Long-Scale Slide Rules

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Introduction

The common slide rule with which we are all familiar has number scales that are about 10 inches long. Calculations with slide rules having this scale length usually can be resolved to a precision of about 3 or 4 digits, or about 0.1% of the result. Smaller slide rules have been made to make it easier to carry a slide rule in a pocket – usually at some cost to precision – but longer scale lengths were developed to give greater precision.

Before proceeding, I will define some terms. I must first define the phrases *number scale* and *long scale.* The number scales on a slide rule that I refer to are the single-cycle scales used for multiplication and division. We often refer to these scales as the C and D scales. Long-scale slide rules are those slide rules with number scale lengths greater than those on the C and D scales on a common 10-inch-scale slide rule. For my convenience, I assume for all slide rules that the left index starts at 100 and the right index is at 1000.

I should also differentiate between *precision* and *accuracy.* There is often confusion about these terms. The distinction is important because precision refers to the number of digits with which a number can be expressed, and accuracy is the exactness[1] of that reading. With a slide rule, one might be able to resolve a precision of 3 digits, but because of poorly laid out scales, the reading might be accurate to only 2 digits. The ability to resolve a calculation on a slide rule to a certain precision is very much affected by the number of tick marks (graduations) that the scale is broken down into and the spacing between the graduations. Long-scale slide rules have space for more tick marks, and thus can be read directly (without interpolation) to greater precision. Indicators of this precision are the values of the first graduations in from the left and right indexes. I call these readings 'tick mark resolution', i.e., TMR. I used the TMR values and the spacing between graduations at the left and right ends of the number scale to make an estimate of the number of digits that could be read by interpolation at either end of the scale. These intervals are generally greater at the left index than at the right index because of the compression of the scale as it runs from left to right. On the standard 10-inch slide rule, the greatest difference in graduation spacing is not at the ends of the scale, but at the number 400. However, for convenience of this study, I have chosen to use the left and right index graduation spacing. Such a space is sometimes large enough to visually resolve it into ten intervals, but often only two intervals can be determined.[2] This part of the study is a bit subjective. When possible, I used the actual cursor to make the estimate of the number of digits that could be read with interpolation. When I had only a photocopy of the scale surface, I used a separate cursor glass with a hairline to make the readings. Typically, the number of digits could not be resolved to a whole number. For example, if the TMR at the right end of the scale was 999 and the space between the next to last graduation and the right index was only large enough to resolve it into two intervals, the next highest reading to 1000 that could be resolved by estimation would be 999.5, or 3.5 digits.

History of Long-Scale Slide Rules – 17th through the early 19th Centuries

The first 'long scale' slide rule may have been the circular slide rule made for William Oughtred by Elias Allen in about 1632. This slide rule was featured on the front and back covers of the *Journal of the Oughtred Society* [15] in March, 1996. Possibly it is the oldest slide rule known. An original example is in the Whipple Museum in Cambridge, England. The disk is about 12.5 inches in diameter, and the number calculating scale has a length of about 30 inches. That makes the number scale on this slide rule about three times longer than the number scale on the common 10-inch slide rule of the 20th century. One can resolve fully 4 digits at the left index and 3 digits at the right index of the Oughtred circular slide rule versus 3.8 and 2.75 digits for the common 10-inch Mannheim slide rule.

Most of the innovations in long-scale slide rule technology came in the period between Oughtred's invention of the circular slide rule and the advent of the modern age of slide rules in the 20th century. My source for much of this early history was the reprint of Florian Cajori's *A History of the Logarithmic Slide Rule* [4]. Most of the early developments in slide rule technology were made in England. It turns out that 'long scale' slide rules were among the first of the innovations that appeared. For instance, Cajori reported that a Mr. Milburne of Yorkshire designed the spiral form of slide rule in about 1650, not long after Oughtred's invention of the circular slide rule. Cajori also found that about this same time, a "Mr. Brown projected Gunter's line into a kind of spiral of 5, 10, and 20 turns." As we shall see later in this paper, the spiral form of long-scale slide rule is probably the most efficient and easiest to use of 'very long scale' slide rules. It is impressive that this format was developed in the very early days of the slide rule. Cajori also reported that in the same period a Mr. Horner put forth a slide

[1] "Exactness" means how close the computed product or quotient is to the correct answer obtained by calculation. *Ed*

[2] Theoretically, these intervals should be logarithmically spaced. *Ed.*

rule "in which the straight edge was replaced by several shorter rules". Perhaps this slide rule was the forerunner of the gridiron scales or the segmented slide rules that developed later. Cajori did not report any details of these slide rules, so we do not know what scale lengths were obtained, or how many digits could be resolved in calculations made with them.

The idea of the long-scale slide rule was also promoted by John Ward in 1707. Ward found that pocket slide rules with lengths of "nine inches or a foot long ... at best do but help guess at the Truth". He recommended slide rules two or three feet long to get the accuracy required for gauging. One must recognize that a part of the problem Ward and the gauging profession faced was that the calculating scales on ordinary slide rules were often crudely laid out. It was apparently easier in his time to lay out the scales more accurately on long slide rules.

In 1733, Benjamin Scott described a circular slide rule over 18 inches in diameter with a circular scale having a circumference of 58.43 inches. According to Cajori, Scott was unaware of any forerunners of his work. A few years later in 1748, George Adams engraved a spiral scale with 10 windings on a brass plate 12 inches in diameter. Although it is not known for sure, the scale on Adams' slide rule may have been longer than 16 feet.

In the 1700s, there were also some developments in long-scale slide rule technology in other European countries. In 1717, in Italy, for instance, Bernardus Facini designed a spiral-scale slide rule that has a scale length of about four feet. This rule is illustrated on the front cover of this issue of the *Journal*. Interpolation of readings is aided by the inclusion of vernier-like markings on a band just outside the spiraling scale (see front cover). Readings can be resolved to 4 digits at the left index and 3.5 digits at the right index. The only known example of Facini's slide rule is in the Adler Planetarium & Astronomy Museum in Chicago. In Germany, the scientist Johann Heinrich Lambert had slide rules made (in the 1770s) that had scale lengths of four feet. In 1727, in France, Jean Baptiste Clairaut described a 21-inch-diameter circular slide rule with a large number of concentric circles, one of which was a long-scale number scale. While not known, the length of the number scale could have been greater than five feet. Even more impressive is another long-scale slide rule of Clairaut. In 1720, he designed a rectilinear slide rule laid out on a square of one foot, filled with parallel lines making up a single number calculating scale. Cajori reported that this slide rule had a length of 1500 French feet. I leave it to the reader's curiosity to figure out just how long Clairaut's scale was, but his slide rule appears to be one of the first of the gridiron type.

Later in the 18th century, the Englishman William Nicholson made several contributions to increasing the accuracy of slide rules. In 1787, Nicholson described a straight slide rule having a double line of numbers (2-log cycles) 20 feet in length. According to his design, the scale was broken down into ten segments. This slide rule body must have been about 2 feet long. Nicholson even devised a kind of runner to help with the calculations. Nicholson also developed a circular slide rule having a single line of numbers made up of three concentric circles, and in 1797 he described a 10-revolution spiral slide rule having a total scale length of 41 feet. Cajori shows illustrations of Nicholson's slide rules taken from his writings, but stated that it is uncertain whether any of his slide rules were constructed and sold.

The first mention of a cylindrical slide rule that I found in Cajori's history is one made in 1816 by Hoyau in France. Cajori does not give any details of Hoyau's slide rule, however.

It was not until the 1800s that any important developments in slide rule technology took place in the United States. It appears that the first long-scale slide rule made commercially in the United States was the 8-inch-diameter circular slide rule designed by Aaron Palmer in the 1840s. John E. Fuller improved this slide rule with the addition of a Time Telegraph scale on the reverse and copyrighted it in 1846. Details of this slide rule were reported by Feazel [7]. It sold under the Fuller-Palmer name in fairly large numbers over the next 20 years. An example in my collection has a calculating scale of about 26 inches in length.

The Nystrom Calculator (circular slide rule) appeared in the US in 1851, shortly after the Fuller-Palmer. The Nystrom [13] is elegantly engraved on a 9-1/2-inch-diameter brass disk – somewhat a reminder of the early Oughtred circular slide rule. A kind of vernier is built into the slide rule and cursor markings to aid in the interpolation between graduations. It appears that the vernier can resolve readings to 4 digits at both ends of the scale, since theoretically the vernier can break the space between graduations into 100 parts. However, in practical applications, the resolution at both ends of the scale will probably be much less. Unfortunately, I did not have an example of this slide rule to examine closely. The Nystrom Calculator is very rare, and highly sought after for collections; an example was sold at the Skinner Science & Technology [22] auction in 1997 for $10,350.

Perhaps the first important development in long-scale technology in the US was the cylindrical slide rule patented by Edwin Thacher [23] in 1881. Thacher broke a double calculating scale into 40 segments, each 18 inches long. The cylinder slides inside a sleeve of 20 parallel rods, each having two sections of the double scale matching two sections on the cylinder. The effective length of the Thacher long scale is 30 feet. It can be read (with interpolation) to 5 digits at the left index and 4 digits at the right index, an improvement of about 1.2 digits over the common 10-inch slide. According to Cajori, both J.D. Everett (a Scottish 19th century maker of slide rules) and Amédée Mannheim (the most influential of French slide rule designers in the 19th century) also designed slide rules of the Thacher type before Thacher's model became popular.

Clairaut's and Nicholson's ideas of breaking the calculating scale into many parts (segments) of equal length and laying them out in parallel lines was also proposed in different forms by Everett, Hannyngton, Scherer, and Proell in the 1800s. The scales on these slide rules are sometimes referred to as gridiron scales. Some have survived to this day. It is interesting to note that the Thacher cylindrical slide rule is essentially a gridiron type with the scales laid out on a cylindrical surface.

Arguably, one of the most innovative long-scale designs was the helix scale laid out on a cylinder. The most widely known of this type of slide rule is the Fuller Spiral Calculator, invented by the Englishman, George Fuller, in 1878. The Fuller Calculator has a scale length of almost 42 feet. According to Cajori, other cylindrical rules with spiral scales were also designed by G.H. Darwin and R.H. Smith near the end of the 19th century.

One other long-scale slide rule innovation reported by Cajori takes the form of a tape that is taken up on a spool or spools. The "idea is to place the logarithmic line upon continuous metallic tapes, wound from one roller or spool upon another as in instruments by Darwin's designs and B. Tower's". Cajori provides no details on this type of slide rule, but later in this paper I will show details of this type of slide rule developed by J.R. Paisley in 1939.

Other innovations were made to obtain greater precision with a slide rule without increasing the scale length. For instance, vernier attachments were made for the cursor. Cajori reports that this was done as early as 1851 by J.F. Artur in Paris, and later by O. Seyffert in Germany. "Perhaps the best known is the Goulding cursor, which allows the space between two consecutive smallest divisions of a rule to be divided into ten equal parts." For those with an interest in how the Goulding vernier cursor works, Pickworth [17 and 18] shows a drawing of the Goulding cursor and describes its function.

Another idea to improve precision of slide rule calculations that deserves mention here is the magnifying cursor. That cursor also serves to obtain more precise results without increasing the scale length. Many of the 20th century slide rule makers offered magnifying cursors or magnifying attachments to cursors as accessories.

An Analysis of Long-Scale Rules

The following describes details of the long-scale slide rules developed since Oughtred's early invention of the slide rule. The focus is on the single-cycle calculating scales used on a slide rule for multiplication and division, the type of calculating scale that we commonly designate as the C or D scale on the modern slide rule.

In the 20th Century, a number of different approaches to making long-scale slide rules were employed in one form or another. One of the most common was to lay out 20-inch calculating scales on a longer slide rule body. We all know the 20-inch Mannheim slide rules by K&E, Dietzgen, Faber, and others. These slide rules are straight in format. Even longer slide rule calculating scales were obtained on cylinders or disks, examples being the Thacher, Fuller, and Gilson Atlas slide rules.

Tables 1 and 2 summarize the results of my study. I found four different formats for laying out the calculating scales: 1) linear or straight; 2) circular; 3) cylindrical, and 4) tape; and five different scale configurations: 1) sectional; 2) circular; 3) spiral; 4) helix, and 5) sawtooth. Each table lists these details for each of the slide rules, and also shows the tick mark resolution (TMR) at both ends of the number calculating scale, and my estimate of the number of digits that can be resolved by interpolation.

Slide Rules with Linear Formats

Table 1 shows that the ordinary Mannheim slide rule has a scale length of about 10 inches. The results can be resolved to 3 digits at the left index, and 2.5 digits at the right index. With interpolation, the number of digits that can be resolved is 3.8 at the left index and 2.75 at the right index. If the length is doubled, as for instance on any of the 20-inch K&E log log duplex slide rules, the resolution is 3.5 at the left index and 2.8 at the right index, and the number of digits that can be resolved by interpolation is 4 at the left index and 3 at the right index. The resolution improves by 20% or less when the scale length is doubled, so it can be seen that large increases in scale length are needed to significantly increase the number of digits that can be resolved on a slide rule.

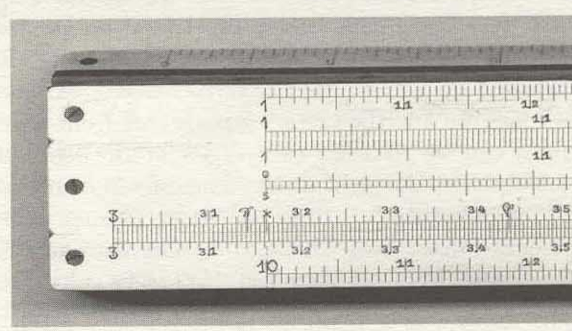

Figure 1. Detail of the left index of the Nestler 27a Precision Slide Rule.

Further to increase the scale length without increasing the length of the body of the slide rule, some slide rule makers developed rules with segmented scales. For instance, the Nestler [14] Precision No. 27a slide rule breaks a pair of calculating scales into two segments, each 20 inches in length. The first segment runs from 100 to $100 \times \sqrt{10}$, and the second segment runs from $100 \times \sqrt{10}$ to 1000. On the Precision slide rule, there is a pair of 1st segments at the upper margin of the slide and a pair of 2nd segments at the lower margin of the slide. Figure 1 shows the scale arrangements at the left end of this slide rule. The effect of this arrangement is to give a slide rule of 40-inch length. This slide rule can be read to 4 digits near the left index and to 3.5 digits near the right index.

Table 1. Slide Rules with Straight Scales

Maker	Model	Configuration	Approx. Date	Effective Scale Length inches	Tick Mark Resolution Left Index units	Tick Mark Resolution Right Index units	Number Digits With Interpolation Left Index digits	Number Digits With Interpolation Right Index digits
STRAIGHT SCALES		Segments						
K&E	Mannheim	1	1910's	9.8	101	995	3.8	2.75
Merrifield	Gunter Rule	1	1850's	11.2	102	990	3.8	2.75
K&E	Mannheim 4045	1	1910's	15.8	100.5	995	4	3
K&E	LL Duplex #4081-5	1	1920's	19.7	100.5	998	4	3
K&E	Cox Duplex #4079	1	1900	19.7	100.5	998	4	3
ACE	The "ACE" Rule	1	1930's	20.0	101	998	4	3
Scofield	Engineer's Slide Rule	1	1905	22.0	101	995	4	3
SEGMENTED SCALES		Segments						
Favor, Ruhl & Co.	M -D 7" Dual 10-5	2	1940	9.8	101	995	3.8	2.75
Nestler	Precision Slide Rule #27/1	2	1910	11.8	101	995	3.8	2.75
Nestler	Precision Slide Rule #27/8	2	1910	19.7	100.5	998	3.8	3
Dietzgen	Log Log Decimal Trig #1741	2	1960's	19.7	100.5	998	3.8	3
Faber-Castell	Novo-Duplex #2-83 & #2-83N	2	1960's	19.7	100.5	998	3.8	3
Unique	10/20	2	1960	20.0	100.5	998	3.8	3
Nestler	Precision Slide Rule #27/9	2	1910	39.4	100.2	999	4.0	3.5
Post	20-inch Mannheim	2	1920's	39.4	100.2	998	4.0	3.5
Unique	Pioneer Long Scale	4	1960's	44.0	100.05	999.5	4.5	3.5
Anderson	Improved Slide rule	4	1910	47.2	100.2	999	4.5	3.5
Hemmi	#201	4	1960's	80.0	100.1	999.5	4.5	3.75
Hemmi	#200	6	1930's	96.0	100.1	999.5	4.5	3.75
GRIDIRON SCALES		Segments						
Hannyngton	Hannyngton's - small Extended	5	1866	62.5	101	999	4.5	3.5
Gilson	Pocket Slide Rule	14	1915	70.0	100.1	999	4.5	3.5
Cooper	100-Inch	20	1900	100.0	100.2	999	4.5	3.5
Hannyngton	Hannyngton's - large Extended	8	1866	125.0	100.2	999	4.5	4
Cherry's	Calculator	20	1880	200.0	101	995	4.8	4
Calculigraph	Australian Slide Rule	22	1909	216.0	na	na	4.8	4
Nicholson	long scale	10	1797	240.0	100.1	980	4.8	4
SAW TOOTH SCALES		Segments						
Lurie	Precision	1	1910	20.0	110	980	3.0	2.8
Richardson	Pyramid #1898	20	1915	200.0	100.1	999	4.8	3.8
CHARTS / TABLES		Segments						
MacMillan	Table Slide Rule	20	1925	na	100.23	997.7	4.4	3.4
Goodchild	Mathematical Chart	100	1900's	650.0	100.1	999.8	4.8	4
LaCroix and Ragot	Graphic Table book	1000	1938	4370.0	100.01	999.99	6	5.8

There is also a 20-inch (German made) Post slide rule with 40-inch scales made up of two segments. This slide rule is interesting because the scale sections on the slide are inverted - something like the CI scale on a modern slide rule. It was probably made in the early 1900s, but I do not have any references to support the date. This slide rule is in a private collection.

The 10-inch Unique Pioneer Long Scale and the 20-inch Hemmi No. 201 slide rules take the segmented scale innovation to another level. Both of these slide rules break the calculating scales into four pieces each, the resulting scale length being 40 inches for the Unique Pioneer and 80 inches for the Hemmi No. 201. The C scale is broken into four equal-length segments on the slide and four matching D-scale segments on the lower stator. For the Hemmi No. 201, the number of digits that can be resolved is further improved to 4.5 at the left index and 3.75 at the right index. However, it begins to get a bit tricky in deciding on which scale to read the result. One either calculates the approximate result in one's head, or resorts to making a calculation with a normal pair of C and D scales before using the segmented C and D scales.

The Hemmi No. 200 is a 16-inch duplex slide rule that breaks the scale into even more segments. It breaks the C and D scales into six sections each, giving an effective 96-inch scale length. The number of digits that can be resolved is identical to those on the 80-inch Hemmi No. 201. As with the Hemmi No. 201, one must be adept at calculating the approximate result in one's head, or resort to a normal set of calculating scales to get the approximate result so that one knows on which scale to read the more precise result.

Table 2. Slide Rules with Circular and Cylindrical Scales.

Maker	Model	Configuration	Approx. Date	Effective Scale Length inches	Tick Mark Resolution: Left Index units	Tick Mark Resolution: Right Index units	Number Digits With Interpolation: Left Index digits	Number Digits With Interpolation: Right Index digits
CIRCULAR with CIRCULAR SCALES		Circles						
Alro	200R	1	1930's	10.2	101	995	3.8	2.8
Controller	110R	1	1960's	11.3	101	995	3.8	2.8
Loga	Topo	1	1960's	11.8	101	995	3.8	2.8
Calculigraphe	Calculigraphe	3	1910	11.8	101	995	3.8	2.8
Gilson	Midget	1	1920's	11.8	101	995	3.8	2.8
Pickett	Dial-Rule	1	1960's	11.9	101	995	3.8	2.8
K&E	Sperry	3	1910	12.5	101	995	3.8	2.8
Roplex	Circular Slide Rule	1	1930's	13.1	101	995	3.8	2.8
Appoullot	T3 & T4	1	1921	17.8	101	999	4	3
Sexton's	Omnimetre #3	1	1910	19.0	100.5	998	4	3
Sexton's	Omnimetre #'s 1 and 2	1	1910	20.0	100.5	998	4	3
Russian	CTM	1	1975	21.7	100.5	998	4	3
Gilson	Binary	1	1930	25.1	100.5	998	4	3
Gilson	Atlas	1	1930's	25.4	100.5	998	4	3
Fuller/Palmer	Computing Scale	1	1850	26.4	100.2	999	4	3
Troger	37/393/6004	1	1930	28.3	100.5	998	4	3
German	Military	1	1975	29.7	101	990	4	3
Oughtred-Allen	Brass Disk - 316mm in dia.	1	1632	30.0	101	998	4	3
Fowler	Long Scale	6	1927	30.0	101	998	4	3
Pickett	110	4	1960's	50.0	100.2	999	4.5	3.5
Dempster, J.R.	RotaRule - Model AA	4	1930's	50.0	100.2	999	4.5	3.5
Fowler	Long Scale Magnum	6	1927	50.0	100.2	999	4.5	3.5
Fowler	Jubilee Mag. Extra Long Scale	11	1948	79.0	100.2	999.5	4.5	3.75
Sexton's	Omnimetre #6 (Companion)	20	1910	162.0	100.1	999.8	4.5	4
CIRCULAR with SPIRAL SCALES		Spirals						
Appoullot	T3 & T4	2	1920's	26.8	101	999	4	3
Logomat	816 (V & G)	3	1970	43.3	101	999	4	3.5
Facini	spiral	4	1714	48.0	101	999	4	3.5
ALRO	1010 Commercial	6	1940	59.1	100.20	999.5	4.5	3.5
Ross	Precision Computer	25	1930's	360.0	100.1	999.9	4.8	4.5
Gilson	Atlas	25	1930's	393.7	100.1	999.9	4.8	4.5
Nicholoson	Spiral slide rule	10	1797	492.0	105	995	5	4.8
Gilson	Square Atlas	30	1920's	551.2	100.1	999.9	5	4.8
CYLINDRICAL with HELIX SCALES		Revolutions						
J.H. Steward	R.H.Smith Calculator	20	1920's	50.0	??	??	??	??
Otis King	Model "L"	20	1920's	66.0	100.1	999	4.25	3.5
Helice a Calcul	Model No2	50	1930's?	100.0	100.1	999	4.5	3.8
Fuller	Model "I"	20	1878	500.0	100.1	999.8	5	4.5
CYLINDRICAL with SEGMENT SCALES		Segments						
Muto Giken	Kooler Calculator	50	1960's	193.3	100.1	999.9	4.8	4
Thacher	Cylindrical Slide Rule	40	1890's	360.0	100.1	999.8	5	4.2
Loga	15-meter	60	1930's	590.6	100.1	999.9	5	4.5
TAPE SLIDE RULES		Segments						
Paisley	Calculator, Model A	1	1939	20.0	101	995	3.8	2.8

Slide Rules with Gridiron Formats

Gridiron slide rules break the calculating scale into a series of sections laid out one below the other. Usually they have some sort of sliding piece placed over the scales that facilitates the calculations. The earliest of the gridiron slide rules found was the ten-segment, 120-inch total length slide rule attributed to William Nicholson. It was described in the 3rd edition (1798) of the Encyclopedia Britannica [6]. The Nicholson slide rule (Figure 2) is essentially a ruler consisting of ten parallel scale lines with a beam compass-like device that slides over the surface of the rule. It had the longest scale length of the gridiron type found, about 20 feet long. This slide rule can be read to nearly 5 digits at the left index, and 4 digits at the right index. It is uncertain whether any of these calculating rules were made.

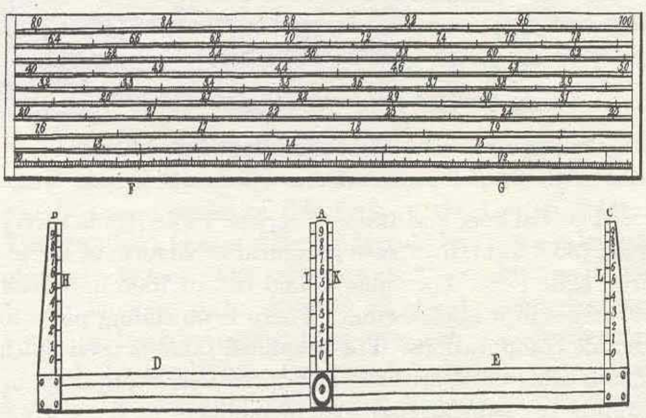

Figure 2. Drawing of the Nicholson 'Gridiron Type' slide rule [6].

Another gridiron slide rule is the Hannyngton Extended slide rule, which was first made in the mid 1800s. Two sizes of this slide rule are known, the smaller having five segments and a total length of 62.5 inches, and the larger one having eight segments and a total length of 125 inches, twice the length of the smaller version. The extra length improves the resolution only slightly. The segmented scale is repeated much like the two-cycle A and B scales on a modern slide rule. This helps facilitate the calculation by keeping the runner from falling off one end of the scale. Examples of both sizes of the Hannyngton gridiron slide rule are known in collections, but surviving examples are very rare. According to Pickworth [17], the gridiron slide rule developed a little later by Cherry in 1880 used a transparent upper sheet with the same segmented scale as laid out on the lower sheet. Indices at all four corners of the transparent sliding sheet facilitate the calculation without the need of repeating the scale on the lower sheet. Another gridiron slide rule contemporary with the Hannyngton and Cherry slide rules was Proell's Pocket Calculator. Not much is known about this gridiron slide rule except that the scale on the transparent sliding sheet was inverted, much like the CI scale on a modern slide rule.

An instruction manual issued in 1909 by the Kolesch Co. [16] listed a gridiron slide rule called the Calculigraph or Australian Slide Rule. The scales are printed on 9 in.×11 in. cardboard in two ranks of 22 parallel segments to form a two-cycle calculating scale. A sliding transparent 'bridge' with a one-cycle segmented scale is used to make the calculations. According to the advertising material, this slide rule had a maximum error of 1 in 5000, or about 4.5 digits precision. No other details of this slide rule are known.

The Gilson Slide Rule Company is better known for its circular slide rules, but it appears that very early in this company's existence (ca. 1915) it made a linear pocket slide rule that broke the calculating scales into 14 sections. An advertisement in the instruction manual for the Richardson Direct Reading slide rule, *The Slide Rule Simplified* by Richardson & Clark [19] shows the Gilson Pocket Slide Rule having 14-section calculating scales for a total scale length of 70 inches. The scales are printed on "heavy water-proof Bristol", a cardboard-like material. The price was 50 cents. Readings on this slide rule can be resolved between 4.5 digits and 3.5 digits with interpolation. The only mention of this slide rule that I have been able to find is in the Richardson and Clark instruction manual. It is uncertain whether any examples have survived.

The Cooper (20 segments, 100 inch length) is an interesting variant of a gridiron slide rule. This slide rule was described in detail in an article by Bennett [3]. A single 20-section scale is laid out on a white celluloid sheet laminated to a mahogany board, with the scale on 20 parallel lines, each 5 inches long. The corners of the block of scales are marked with special indicator marks labeled with the number 100. A separate clear celluloid sheet with matching scales slides over the calculating scale. The calculations are made using the appropriate corner indicator mark and a weighted pointer that freely slides over the clear celluloid sheet. It can be read with interpolation to between 4.5 and 3.5, similar to that of the Gilson pocket slide rule. This appears to be one of the easiest of the long-scale slide rules to use. According to Bennett, the Cooper slide rule can be worked rapidly with no ambiguity in reading the answer. The Cooper was British made, perhaps in the 1920s and 1930s. This slide rule is very rare, with only two examples known in collections.

Slide Rules with Sawtooth Formats

One more interesting idea that improves slide rule precision parallels the idea of the vernier cursor. This method incorporates the "vernier" into the calculating scale by a kind of sawtooth arrangement. Babcock [1] reported on two different approaches. One by A.N. Lurie developed in 1910 uses diagonal lines drawn from the bottom of one scale division to the top of the next. This diagonal line, in combination with a series of horizontal crossbars on the cursor, allows the user to divide the space between the divisions into ten parts. Lurie applied his method to an ordinary 10-inch Mannheim slide rule, but Richardson employed a similar concept to a grid-iron type scale. Richardson (ca. 1918) used a method designed by a Chinese man, Yu Wang, whereby a kind of tent or triangle is laid out between graduations. In Wang's design, five parallel horizontal lines are drawn within the triangle. The reading of the cursor hairline is then determined by where it crosses the point of intersection of one of the horizontal lines and one of the diagonal lines. Figure 3 shows Richardson's slide rule. This same concept was employed on the Appoullot [19] circular slide rule, but in this case the "tent" between graduations is formed by ten short lines drawn normal to the scale direction. In this case, the "tent" is located in the space outside of the calculating ring where there is more room. The Appoullot slide rule is also interesting because it incorporates a spiral scale.

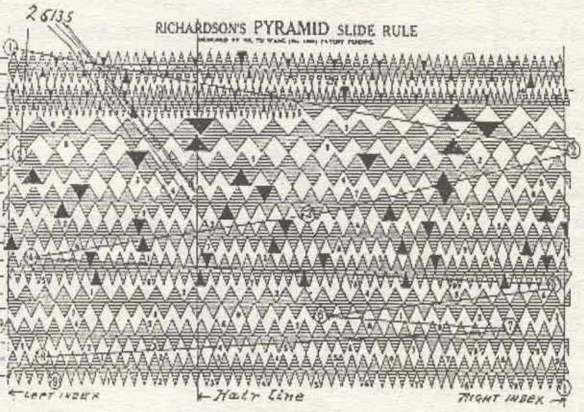

Figure 3. Detail of Richardson's Pyramid Slide Rule.

Charts and Table Slide Rules

The Goodchild and the LaCroix and Ragot calculating charts are interesting variations of the gridiron type slide rule. The Goodchild Mathematical Chart and its accessory Triangular Rule were sold by K&E [10] for a short time in the early 1900s. The chart breaks the number scale down into 100 parallel segments on a single folded card stock sheet. The total scale length is over 54 feet. Each line is numbered at the beginning and end with the first two digits of the mantissa. The balance of the logarithm is represented by the distance along the segment. Every fifth graduation along the segment is labeled with the number represented in the logarithmic scale by the particular segment and distance. The Triangular Scale acts like a bridge to enable the calculations. One side has scales and a slide to add (or subtract) the first 2 digits of the mantissa of the numbers in the operation. This gives the initial 2 digits of the line on which the result will be found. The other two sides each have a series of equally spaced graduations and a very short slide that is used as an index marker to keep track of the distance of the reading from the left edge of the column. Readings can be resolved to nearly 5 digits at the left index and 4 digits at the right index. Figure 4 shows the triangular rule on a reproduction of the Goodchild chart. A few of the Triangular Rules are known, but I have been unable to turn up an original example of the Goodchild chart.

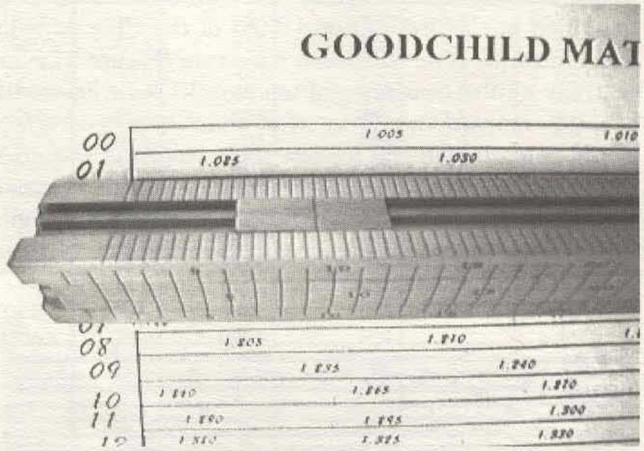

Figure 4. The Goodchild Triangular Scale on a reproduction of the Goodchild Chart.

The LaCroix and Ragot Graphic Table [12] is a very long (36.4 feet), five-place graphical table form of a grid-iron slide rule. The table is laid out in 1000 lines over 40 pages in a book format. There is no sliding piece to enable computations. The graphical table is used much like a table of logarithms, the operations being done by adding (or subtracting) logs of numbers in the normal way. The advantage of this table is that it is much more compact than a conventional five-place table of logarithms. Readings can be resolved to 6 digits at the left index and 5.8 digits at the right index.

One other chart-type slide rule deserves attention. The MacMillan Table Slide Rule [2] is unique among slide rules in that it takes the form of a table of discrete logarithms, not an analog scale of logarithms. There are four tables laid out on card stock, one each for number, logarithm, sine and tangent operations. Each table has two cycles of data in a 201-line by 20-column format. Each line is labeled with the first two digits of the number, and each column is labeled with the third digit. The calculations are performed using cardstock slides which have a matching format, but with only half the width for one cycle. Slides are provided for multiplication, division and square roots. Two slides are needed for each operation. Each card has notches in the upper left and lower right corners to facilitate calculations. Multiplication operations, for example, are made by setting the multiplicand in one of the notches of one slide, and the multiplier in a notch on the second slide. The product is read on the main table in the remaining notch on the second card. Results can be improved by interpolation.

Circular Slide Rules with Circular Scales

The circular slide rule is probably the most common of the long-scale slide rules of the 20th century. Following the innovations of the earliest slide rule makers, many different makers of circular slide rules emerged in the 1900s. I found single-ring circular slide rules having scale lengths from 10.2 inches (Alro No. 200R) up to 29.7 inches (an East German military slide rule). The Alro is compact enough to fit in a coat pocket, but the German slide rule needs a briefcase-sized satchel to carry around. These slide rules could resolve from 3.8 to 4 digits at the left index, and 2.8 to 3 digits at the right index. Circular slide rules with scales on a series of concentric rings had scale lengths ranging from 11.8 inches (Calculigraphe; 3-rings) to 13.5 feet (Sexton's Omnimetre No. 6; 20 rings). These slide rules could resolve from 3.8 to 4.5 digits at the left index, and 2.8 to 4 digits at the right index. The Calculigraphe is in the form of a pocket watch, whereas the Omnimetre is laid out on a large cardstock sheet.

Circular Slide Rules with Spiral Scales

I found ten different makers of circular slide rules with spiral scales. The Gilson Atlas was probably the most widely known make of this type in the United States. The standard Gilson Atlas slide rule has a scale length of nearly 33 feet. The spiral winds 25 revolutions on an 8-inch-diameter disk. As with all spiral slide rules, the major problem is keeping track of which scale revolution to find the result on. For any calculation, the cursor hairline will cross the calculation scale in 25 places. If one is adept mentally, the calculation can be approximated in one's head, and the appropriate scale crossing can be selected. However, Gilson did not leave this to chance or mental error. The Atlas has an extra ring at the outer edge of the disk that contains one complete calculating scale. The operator first makes the calculation on this outer ring to obtain the result to 3 to 4 digits, and then repeats the calculation on the spiral scale to get the result to 4.5 (right index) to 4.8 digits (left index). Note that with the spiral scale the precision of the readings is nearly the same at both ends of the scale. This is the result of the increasing diameter of each winding, which results in more space for graduations as the scale winds its way to the outer edge. It is more pronounced on spiral scales with large diameters and large numbers of windings. Gilson also made an early version of the Atlas, sometimes referred to as the 'square' Atlas, that has 30 windings, and a scale length of nearly 46 feet. This slide rule could resolve nearly 5 digits at both ends of the scale. The 'square' Atlas has a scale length even longer than the scale lengths of the Fuller and Thacher cylindrical slide rules that will be discussed later.

The Ross Precision Computer has a spiral calculating scale, much like that of the common Atlas slide rule. It has 25 windings, and the scale length is 30 feet. It can be read to about the same precision as the Atlas. The Ross, however, takes a little different approach to obtaining the approximate answer. It has a rectilinear slide rule attached to a radial arm on the pivot point. One first makes the calculation on the straight slide rule. The position of an arrow on the slide then lines up with the appropriate winding on the spiral scale. Examples of the Ross slide rule are quite scarce – with maybe ten to twenty examples in collections. Many of these are in poor condition because of the unstable metals used in their manufacture.

The Appoullot [21], Logomat [20], and Alro spiral slide rules are smaller in diameter and have many fewer windings than the Atlas and Ross slide rules. The precision of the readings made with these rules is about one digit less.

Cylindrical Slide Rules with Helical Scales

The Fuller is one of the most widely known of the cylindrical slide rules made. Fuller started making these slide rules in the 1880s, and continued making them right up until the electronic pocket calculator brought an end to slide rule use in the early 1970s. It has been well described in a 1994 volume of this *Journal*[9]. The Fuller has a scale that winds around the cylinder 20 times to give a total scale length of about 42 feet. One can resolve the readings to a precision of 5 digits at the left index and 4.5 digits at the right index, not quite as good as for the Gilson 'square' Atlas, but a little better than the common Gilson Atlas slide rule. Other production cylindrical slide rules with helix scales include the well-known Otis King pocket cylindrical slide rule, and the less common R.H. Smith Calculator. The Otis King model 'K' has one double scale with forty windings and a second single log cycle scale with twenty windings about a nominal 1-inch-diameter cylinder. A sliding cursor sleeve facilitates the calculations. The Otis King scale length is 5.5 feet, and it can be read with a precision of about 4.25 digits at the left index and 3.5 digits at the right index. Examples of the Otis King are very common. They were made from the 1920s until the early 1970s. They are perhaps the first of the cylindrical slide rules to make it into a beginners's collection.

The R.H. Smith Calculator was described by Weinstock [24] in a previous issue of this *Journal.* A description of this cylindrical slide rule also appears in the 11th edition (1910) of Pickworth's book [17]. Weinstock's RHS slide rule has a cylinder diameter of 1/2 inch and a scale length of about 40 inches, in contrast to the 3/4-inch-diameter cylinder and 50-inch scale length reported in the Pickworth book. There must have been at least two different models of the Smith Calculator. The cursors are also different for these two models, the Weinstock version having two brass rods, much like the Fuller cylindrical slide rule, whereas the Pickworth version has an actual sliding cursor something like that of the Otis King, only much shorter.

I have one other cylindrical slide rule with a spiral calculating scale in my collection that deserves mention. It is marked Helice a Calcul (slide rule, in French) No. 2, and was made by A. Lafay of Neuville S/Saone. The spiral scale winds fifty times around a 1.6-in.-diameter tube to give a total scale length of 100 inches and a precision of 4.5 digits at the left index and 3.8 digits at the right index. A sliding celluloid sleeve and three celluloid cursors facilitate the calculations. Figure 5 shows the Lafay slide rule. I have been unable to find any other information on this slide rule and its maker.

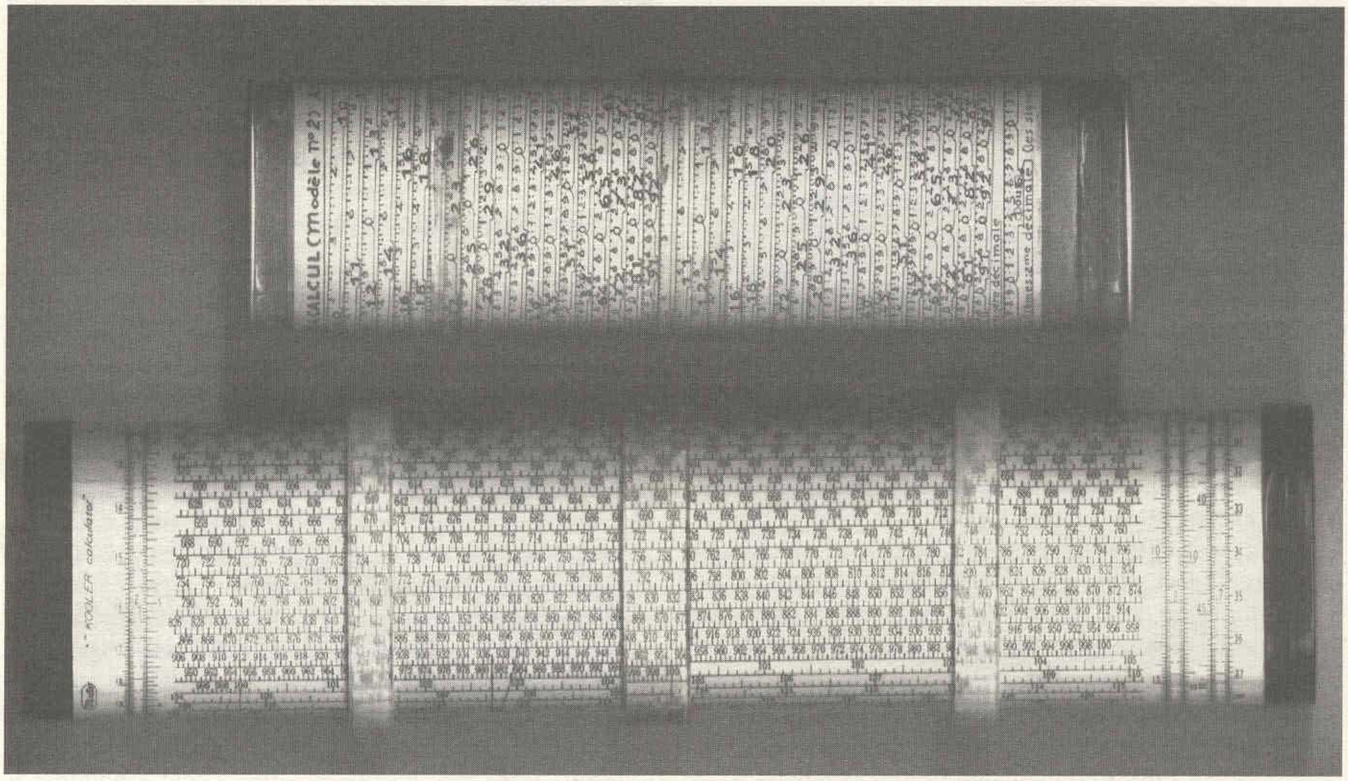

Figure 5. Two cylindrical slide rules: bottom, Kooler Calculator; top, Lafay Helice a Culcul.

Cylindrical Slide Rules with Linear Segment Scales

The most widely known of the cylindrical slide rules with linear segment scales is the Thacher Calculator patented by Edwin Thacher in 1881. The Thacher slide rule has been described numerous times in the *Journal of the Oughtred Society,* so I will not go into much detail here. It is essentially a double calculating scale that slides inside a cage made up of twenty rods. Each rod has a pair of scales that match appropriate sections of the calculating scale on the cylinder. This slide rule has an effective length of 30 feet, and a precision of 5 digits at the left index and 4.2 digits at the right index. The Thacher slide rule was made from the 1880s into the 1940s. Many examples of this slide rule are known, but it remains one of the more costly of all slide rules.

The Loga cylindrical slide rule is similar to the Thacher in its operation, except that the cylinder is fixed and the scales, on a sleeve, slide on the cylinder. The sliding sleeve has a length about half that of the cylinder, and the segments make up a single scale. This is a Swiss-made calculator that has its origins in the early 1900s. It came in several different diameters and scale lengths. Similar versions appear to have been marketed by Nestler, a German maker of slide rules. The Loga cylindrical slide rule in my collection is a 15-meter version, which has a scale length of about 49 feet, and a precision of 5 digits at the left index and 4.5 digits at the right index.

One other cylindrical slide rule of this type was brought to my attention by Rodger Shepherd. That is the Japanese Kooler Calculator by Muto Giken (Figure 5). It is a modern version (perhaps 1960s) with fifty double scale segments laid out longitudinally on a 1.85-inch-diameter by 11.5-inch-long tube. The effective scale length is about 16 feet. It can be read to a precision of 4.8 digits at the left index and 4 digits at the right index. This is the only example of this slide rule that I have seen.

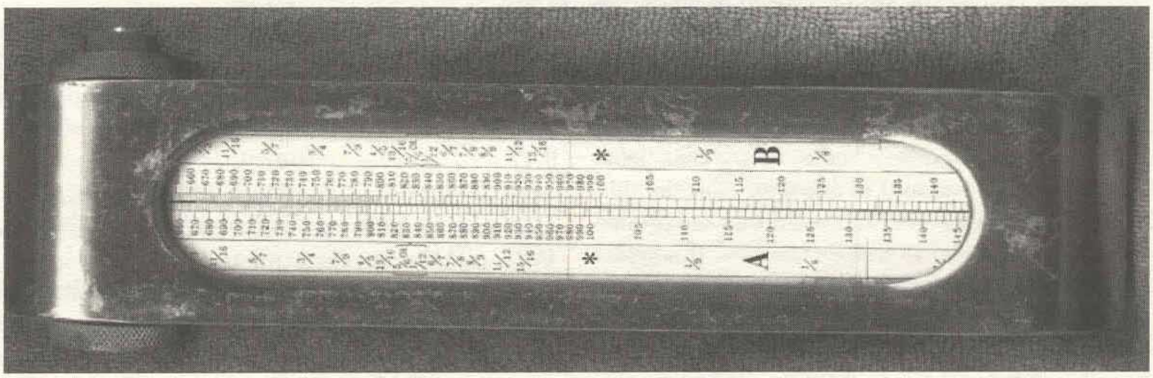

Figure 6. The Paisley Calculator.

Tape Slide Rules

The only long-scale slide rule that I found with a tape format was one copyrighted by Paisley in 1939. This slide rule has two continuous scales on side-by-side ribbons that wrap around spools at each end of the device. The scales are read through a window in the case. The Paisley slide rule was described briefly by Feely [8]. The scales on the ribbons are positioned relative to each other by turning knurled knobs at one end of the device. The operation is very straightforward. The Paisley Calculator (Figure 6) has a scale length of 20 inches and reads with a precision of 3.8 digits at the left index and 2.8 digits at the right index. I have seen only two examples of this slide rule.

Some Observations and Conclusions

In Figure 7 I have plotted the number of significant digits that could be estimated with the slide rules listed in Tables 1 and 2 against the scale length. There appears to be a good linear correlation when the number of digits is plotted versus the log of the scale length. One can see that differences in the precision between the left and right indices are smaller as the scale length increases. Figure 7 also shows that spiral scale improves the resolution at the right index. This is the result of the increasing length of the spiral scale as it winds its way towards the outer rim of the disk. The same argument could probably be made for the circular scales made up of many circular rings; however, none of that type of slide rule reviewed had the right combination of the disk diameter and number of rings to take advantage of that outcome. Figure 7 also shows that the law of diminishing returns is working against improving the precision of slide rules. To design a slide rule for 6-digit precision, one would have to increase the length of the longest slide rule scale known from about 50 feet to perhaps 750 feet. This would make for a truly monumental slide rule, perhaps on the same scale as that of the first digital computer.

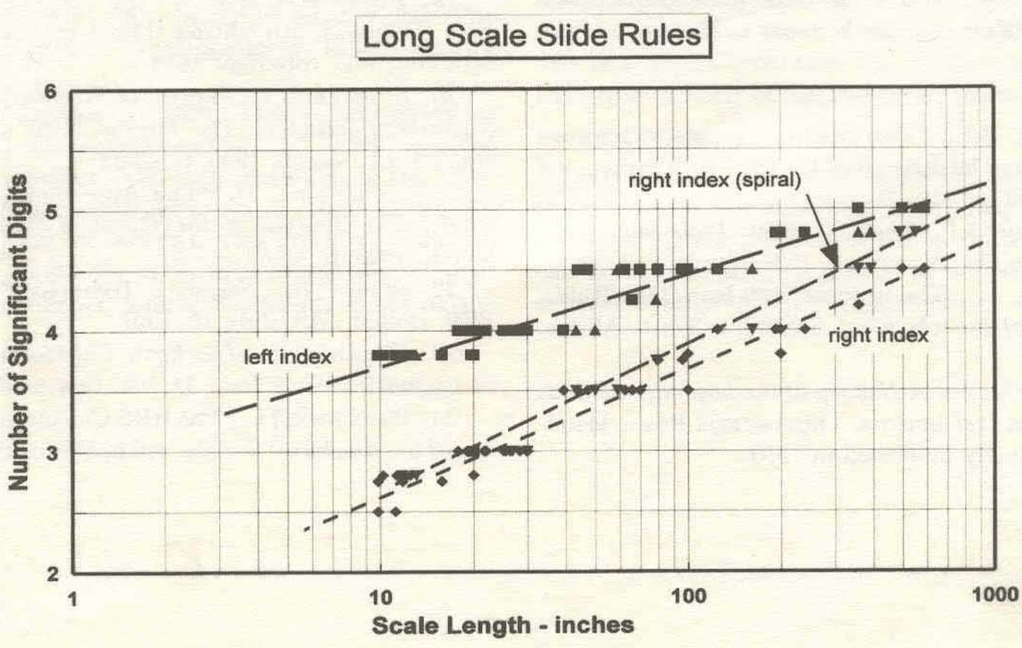

Figure 7. Plot of the number of significant digits versus scale length.

Probably one of the other most important observations of this study is that the Gilson Atlas circular slide rule can be read with nearly the same precision as the Thacher cylindrical slide rule, and it can be done much easier and with considerably less complexity and much less cost. The Thacher slide rule has many components and parts, a rather delicate paper scale surface that deteriorates with time and use, and it takes up considerable space on a desk. One has to get used to finding the proper scales on the sleeve of rods, as well as those on the cylinder. Both have to be rotated independently of the other. In contrast, the Atlas slide rule is a simple disk with a fairly durable enameled scale surface. All scales are fully visible at a glance. It is simple to operate, and fits easily in a desk drawer when not in use. The Atlas slide rule cost $9 in the 1952 edition of the Dietzgen catalog [5], whereas the Thacher Calculating Instrument cost $70 in the 1944 40th edition of the K&E catalog [11]. Perhaps it is not just a coincidence that the Thacher slide rule went out of favor at just about the time that the Atlas slide rule was introduced. Of course, if you want to make a lasting impression on your friends and neighbors, you show them your Thacher slide rule rather than your Atlas slide rule. Nonetheless, I consider both of these slide rules important to any slide rule collection. They represent the best in slide rule precision technology.

Acknowledgements

I should like to express my appreciation to those who sent me information on long-scale slide rules. These include: Bob Otnes, Rodger Shepherd, IJzebrand Schuitema, Bobby Feazel, Collin Barnes, Bob DeCesaris, Dick Rose, Hermann van Herwijnen, Dick Lyon, Thomas vander Zijden, Andreas De Man, Mike Gabbert, Francis Wells, Wayne Lehnert, and Tom Wyman. I also wish to thank the Adler Planetarium & Astronomy Museum for their permission to reproduce the photograph of the Facini circular slide rule on the cover of this issue of the *Journal.*

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The plate (only) from a Ross Precision Computer. This rule is discussed in the article "Long-Scale Rules" by Dr. Chamberlain in this issue. The plate is about 8.3 inches in diameter, and is made from a zinc-like material which tends to oxidize, giving it a smokey appearance. The oxidation makes it difficult for paint to adhere to the material. The plate was probably made using a photo-engraving process.